Semantic Pollution of Proof Systems

Robin Martinot

Universiteit Utrecht

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- What is a faithful proof-theoretic formalization?
- How can we understand differences between proof systems conceptually?
  - Simplicity, explanatoriness, purity (Martinot, 2022), depth, ...

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- "Because of the proof-theoretical nature and the expected generality" (Avron, 1996)
- Value of soundness and completeness proofs?
- So that proof systems reflect ways of reasoning

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Side question: Is reflection of model theory in proof rules compatible with inferentialism? (Read, 2015)

Previous literature

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Two conceptions of syntactic purity:

1. *Strong syntactic purity.* "A sequent calculus should be independent of any particular semantics. One should not be able to guess, just from the form of the structures which are used, the intended semantics of a given proof system" (Avron, 1996).

2. *Weak syntactic purity.* A sequent calculus should not make use of explicit semantic elements (Poggiolesi, 2010).

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- *Semantically polluted*: external calculi (not every element of the calculus has a formula interpretation)

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Unsatisfactory: translatability $\not\Leftrightarrow$ no semantic content

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Question: What is the boundary for considering representation of semantic content 'pollution'?

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1. General pollution of proof systems
2. Imperfect measures for strong syntactic purity
3. A better measure for weak syntactic purity

General pollution of proof systems

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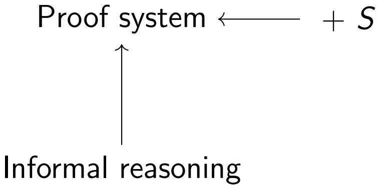

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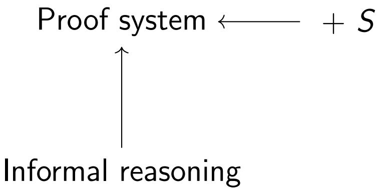

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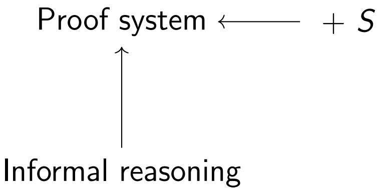

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- Untranslatable: adding $\Box$ to $\mathcal{L}_{\mathsf{PL}}$, adding xRy to $\mathcal{L}_{\mathsf{K}}$
- Translatable: adding $\in$ to $\mathcal{L}_{\mathsf{PA}}$, adding $\times$ to $\mathcal{L}_{\mathsf{ZFC}}$

General pollution of proof systems

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Proof system $\longleftarrow$ $+\ S$

$\uparrow$

Informal reasoning

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- Translatable: adding $\in$ to $\mathcal{L}_{\mathrm{PA}}$, adding $\times$ to $\mathcal{L}_{\mathrm{ZFC}}$

Not pollution: adding $\subseteq$ to $\mathcal{L}_{\mathrm{ZFC}}$, adding $@_a$ to $\mathcal{L}_{\mathrm{K}}$

From a proof system to semantic truth (I)

> A proof system is *semantically polluted*$_1$ if its inference rules for the logical constants are categorical.

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- Consider a proof system and its relation $\vdash$ and a model theory with a relation $\vDash$
- Carnap's Problem: are there interpretations that give a non-standard meaning to logical constants?
- Formal interpretation of 'guessing' or 'inferring' a semantics

From a proof system to semantic truth (I)

CPL is not categorical:

$$\vdash \varphi \Leftrightarrow \vDash \varphi \text{ (wrt all admissible valuations in } V)$$

Let $v^*(\varphi) = \top$ (non-standard)

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Fixes for categoricity tell us something about 'closeness' to semantic pollution

- Semantic fixes: constraints on the valuation space
- Syntactic fixes: signed sequents, multiple conclusion sequents, *n*-sided sequents

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- CPL: quickly fixed by 'Non-triviality' and 'Compositionality'
- FOL: Previous assumptions + 'Topic-neutrality' fix categoricity
- IPL is categorical with respect to many semantics

From a proof system to semantic truth (I)

But:

- No good reason to think that proof systems for IPL are semantically polluted, and those for FOL are not
- This method does not directly relate to representation of semantic notions

From a proof system to semantic truth (II)

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CPL. From premises $\dfrac{\{\Gamma_i \Rightarrow \Delta_i\}_{i \in I}}{\Rightarrow \varphi}$ $\quad$ $\dfrac{\{\Gamma_j \Rightarrow \Delta_j\}_{j \in J}}{\varphi \Rightarrow}$ we deduce that φ is true (false) in a model iff for each premise either some $\gamma \in \Gamma_i$ is false, or some $\delta \in \Delta_i$ is true (Hacking, 1979).

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Not applicable to many proof systems — relies on strong assumptions.

Counterintuitive: CPL is polluted; says that characterizing extra syntax in terms of known semantics → pollution.

Taking stock

- The above measures give interpretations of strong syntactic purity, but make the wrong calls intuitively.
- Instead, focus on weak syntactic purity: a sequent calculus should not make use of explicit semantic elements (Poggiolesi, 2010).

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Definition (Semantic element)

1. A model $\mathcal{M}$, the forcing relation $\vDash$, truth values

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Definition (Semantic element)

1. A model $\mathcal{M}$, the forcing relation $\vDash$, truth values
2. Ingredients of a model:
   - Valuation function
   - Elements that help determine truth values (D, W, N, R, $\in$)

(Not just: an untranslatable element)

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1. A representation of A is *explicit* if it is translatable to the proof-theoretic language (R).

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2. A representation of A is *implicit* if this is not the case. The concept can be incorporated within one symbol, represented among multiple symbols, or among all of the syntax ($\top$).

From semantics to proof rules

Can the object language cause semantic pollution?

- Yes: the way semantic elements are referred to is key, not by which expressions
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Other type of pollution: implicit representation & no informal use

Classical propositional calculus

- *Semantic elements*: $0, 1$, $V : P \to \{0, 1\}$, $\mathcal{M} \subseteq P$.
- *Representation*. Most explicit: p refers to an element of $\mathcal{M}$ (object language)
- *Informal use.* Is $\Rightarrow$ used in informal reasoning? (Steinberger, 2011)

$$\frac{\Gamma, A, B \Rightarrow \Delta}{\Gamma, A \wedge B \Rightarrow \Delta}\, L\wedge \qquad \frac{\Gamma \Rightarrow A, \Delta \qquad \Gamma \Rightarrow B, \Delta}{\Gamma \Rightarrow A \wedge B, \Delta}\, R\wedge$$

Syntactically pure systems

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Low level of semantic pollution

Display calculus (for modal logic): introduces $\circ, *, \bullet$

	A	$*X$	$X \circ Y$	$\bullet X$
N_w	$w \vDash A$	$\neg P_w(X)$	$N_w(X) \wedge N_w(Y)$	$(\exists v)(vRw \wedge N_v(X))$
P_w	$w \vDash A$	$\neg N_w(X)$	$P_w(X) \vee P_w(Y)$	$(\forall v)(wRv \supset P_v(X))$

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Note: $\bullet$ is untranslatable to the modal language, so semantically polluted on earlier accounts

Display calculus (for modal logic): introduces $\circ, *, \bullet$

- *Semantic elements* (Kripke semantics): $W, R, \in$
- *Representation.* Most explicit: p refers to $A \subseteq W$ (object language)
  No particular expressions refer to $R, \in$
- *Informal use.* Informal use of $\circ, *$ and $\bullet$ is unclear $\rightarrow$ other type of pollution? (Efficiency, bureaucracy)

$$\frac{\bullet M \Rightarrow A}{M \Rightarrow \Box A}\, R\Box \qquad \frac{A \Rightarrow M}{\Box A \Rightarrow \bullet M}\, L\Box$$

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Low level of semantic pollution

Slightly polluted systems

Classical first-order calculus

- *Semantic elements*: D, V, ϵ
- *Representation:* More pollution by the object language:
  - Terms refer to $d \in D$
  - Predicates/function symbols refer to $A \subseteq D$
  - Instantiations $P(t)$ perhaps refer to ϵ
- *Informal use*: yes (again, sequent symbols are unclear)

$$\frac{\Gamma, \varphi[t/x] \Rightarrow \Delta}{\Gamma, \forall x\varphi \Rightarrow \Delta}\, L\forall \qquad \frac{\Gamma \Rightarrow \Delta, \varphi}{\Gamma \Rightarrow \Delta, \forall x\varphi}\, R\forall$$

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Labelled calculus (for modal logic): introduces $x, y, ...$, :, R

"The labelled method [...] is a semantic method [in that] it imports in its language the whole structure of Kripke semantics in an explicit and significant way" (Poggiolesi and Restall, 2012)

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- *Semantic elements*: $W, R, \in$
- *Representation:* Explicit: $x, y, \ldots$ refer to worlds; R refers to accessibility relation; : refers to $\vDash$
- *Informal use.* Very debatable (also consider intuitionistic logic).

$$\frac{y : A, x : \Box A, xRy, \Gamma \Rightarrow \Delta}{x : \Box A, xRy, \Gamma \Rightarrow \Delta} \; L\Box \qquad \frac{xRy, \Gamma \Rightarrow \Delta, y : A}{\Gamma \Rightarrow \Delta, x : \Box A} \; R\Box$$

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High level of semantic pollution

Similar to the labelled calculus for modal logic are:

1. *Labelled system for intuitionistic logic.*

$$\frac{x \leq y, y : A, \Gamma \Rightarrow \Delta, y : B}{\Gamma \Rightarrow \Delta, x : A \to B} \, R\to$$

Semantically polluted systems

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1. *Labelled system for intuitionistic logic.*

$$\frac{x \leq y, y : A, \Gamma \Rightarrow \Delta, y : B}{\Gamma \Rightarrow \Delta, x : A \to B} \; R\to$$

2. *Neighborhood calculus* (Negri, 2016).

$$\frac{a \in I(x), a \Vdash^{\forall} A, A \lhd a, \Gamma \Rightarrow \Delta}{x : \Box A, \Gamma \Rightarrow \Delta} \; R\to$$

Finally

Tree-hypersequent calculus (for modal logic): introduces /, ;

$(\Gamma / G_1; \ldots; G_n)^\tau := \Gamma^\tau \vee \Box G_1^\tau \vee \ldots \vee \Box G_n^\tau$

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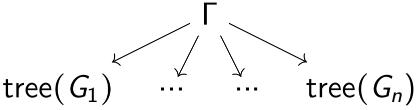

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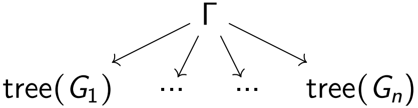

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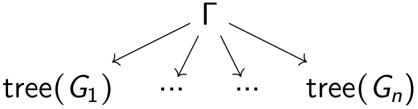

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Seems: low level of semantic pollution

Conclusion

- Semantic pollution$_4$ offers a way to nuance Poggiolesi's weak syntactic purity
- Translatability is important but not decisive in judgements of semantic pollution
- Several further questions arise:
  - Can the object language pollute a calculus?
  - When is something used in informal reasoning?
  - What other types of pollution are there?
  - Apply the method to more proof systems

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