Abstracts “Workshop on Proofs and Formalization in Logic, Mathematics and Philosophy”

September 18, 2022

Tuesday 21 September

9.30-10.10 Andrew Arana, “Meaning and Interpretation in Mathematics”

ABSTRACT Interpretability in logic formalizes the notion of a “dictionary” for translating statements of one theory into statements of another, in such a way that provability is preserved. For instance, translating “geodesic on the pseudosphere” to “Euclidean straight line” leads by way of Beltrami’s theorem to the observation that hyperbolic geometry is interpretable in Euclidean geometry. Interpretability is thus a way to capture a notion of translation that preserves provability. One might seek to take the dictionary metaphor further and ask whether interpretability preserves meaning as well.

In this talk I will address this question by taking into account the constraint on mathematical proof known as “purity of methods”, the proof ideal that says that a proof of a theorem should avoid what is foreign or extraneous to that theorem. Following Hilbert, purity can be understood in terms of meaning, by what belongs to the content of the theorem being proved. Whether a given proof is pure, then, comes down to the meaning of what is proved. It is easy to find algebraic statements interpretable in geometric theories and vice-versa. If meaning is preserved by interpretability, a pure proof of a geometric theorem could employ algebraic statements suitably interpreted in geometric terms. This would be to reject as empty the age-old attempts of mathematicians to develop the autarky of mathematical domains.

This talk’s general thesis is that while interpretability might seem to present difficulties for assessing purity attributions, one must be quite careful in the lessons that one draws from interpretability, and as a result these alleged difficulties are not so clear-cut.

10.20-11.00 Martin Fischer, “HYPE and Cuts”

ABSTRACT In the talk I present sequent systems for the propositional part of HYPE, a logic introduced by Leitgeb for hyperintensional contexts. I will point out some problems for the usual cut-elimination strategies. The similarity to the problem for constant domains in intuitionistic logic suggests that similar solution strategies are applicable. I will follow a strategy used by Kashima and Shimura. By introducing connections as additional information within sequents it is then possible to establish a cut-elimination result.

11.30-12.10 Robin Martinot, "Semantic Pollution of Proof Systems"

ABSTRACT Proof systems and model-theoretic semantics provide different ways of proving results about logics, and soundness and completeness proofs reveal an intrinsic connection between these methods. However, Avron (1996) writes that a requirement of a 'good' proof system is that it should be "independent from any particular semantics". This has become known as syntactic purity of a proof system, as opposed to a semantically polluted one. The value of soundness and completeness proofs seems to come from a certain independence that the syntactic side has from the semantic side. If a proof system is semantically polluted, this may take away from its "proof-theoretical nature and the expected generality" (Avron, 1996). Labeled proof calculi are a standard example of semantically polluted systems, for directly internalizing Kripke semantics in the proof system (see e.g. Poggiolesi and Restall, 2012), but other examples are spread across the literature, including for instance semantic sequents and tableaux and internalized forcing sequents (Poggiolesi, 2010), or the inclusion of neighborhood semantics into the proof system (Negri, 2017).

The goal of this talk is to provide a better conceptual characterization of what semantic pollution is, and to provide and compare formal ways of telling when a proof system is semantically polluted or not. This contributes to a better characterization of what a 'good' proof system can be, and encourages a more nuanced understanding of the distinction between syntax and semantics. The literature distinguishes between a strong and a weak definition of syntactic purity. Strong syntactic purity occurs when a proof system is "independent of any particular semantics" (Avron, 1996). This includes the idea that "one should not be able to guess, just from the form of the structures which are used, the intended semantic of a given proof system". Weak syntactic purity, on the other hand, says that a sequent calculus cannot make use of 'explicit semantic elements' (Poggiolesi, 2010). Poggiolesi argues that strong syntactic purity is too strong, since it implies that basic propositional sequent calculi already have to be declared semantically polluted. Thus, she adopts

weak syntactic purity, where she defines a 'semantic element' as an untranslatable ingredient of a sequent (as compared to a usual original language). This rules out, for example, expressions like xRy in labeled calculi, that explicitly incorporate the notion of possible worlds and the Kripke accessibility relation.

In this talk, we first discuss several informal conceptions of semantic pollution. We suggest that strong syntactic purity might not be too strong after all, by investigating the idea that you can 'guess' the intended semantics just from the rules of a proof system. As Hacking (1979) notes, independently deducing the semantic meaning of inference rules relies on strong semantic assumptions, that limit any real guessing process. Thus, we explore weaker but also more formal understandings of 'guessing' a semantics. We also provide a conception of a 'semantic element' so that it, in a sufficient way, has "an evident connection with truth or warranted assertibility" (Dummett, 1998).

Based on these ideas, we move to possible ways to formally characterize semantic pollution. For weak syntactic purity, we discourage the idea that translatability is decisive in the formal description of a semantic element. Instead, we aim to spell out requirements on the formal language in a proof system in order to exclude semantic elements, which also helps us understand better why these elements are excluded. Among such requirements is the idea that symbols should be able to intermingle sufficiently, and that they should not be able to only say something about the models. We also discuss the view that the level of 'explicitness' of representation of semantic elements is important. Namely, Poggiolesi and Restall (2012) note that elements from Kripke semantics are treated explicitly in labeled systems, but are made implicit in tree-hypersequent systems (reducing the level of semantic pollution). Read (2015) objects that even in tree-hypersequent systems, "the content is still there". We argue that the particular presentation of content does indeed matter for semantic pollution. For strong syntactic purity, finally, we focus on possible formalizations of 'recognizing' semantics from a proof system. For example, the likeness of a proof line to its semantic definition of validity, the degree to which that a syntactic proof simulates a semantic proof, or the way that proof rules determine the semantics of the logical connectives they define, might relate to semantic pollution.

We conclude by considering the implications of this work for various proof systems, and we reflect on whether our measures of semantic pollution seem to interact with any other philosophical properties.

14.00-14.40 Jeremy Avigad, "Proof Systems in Computer Science"

ABSTRACT I will describe various types of proof systems that arise in computer science, including interactive proof systems, cryptographic proof protocols, and proof calculi and exchange formats for automated reasoning. I will also explore some of the goals and constraints that the designers of such systems have to answer to.

14.50-15.30 Johannes Korbmacher, "Hyperintensional Proof Theory"

ABSTRACT In this talk, I'll investigate how to develop "good" proof systems for hyperintensional logics. Odintsov and Wansing define a logic L to be hyperintensional iff at least one of its operators doesn't respect L-equivalence. Hyperintensional logics arise in various places in philosophical logic, ranging from logics of metaphysical grounding to deontic logics of permission and obligation.

While it's typically straightforward to obtain *some* sound and complete proof system for a given hyperintensional logic, these systems are typically not particularly informative. The question that we'll be tackling is how to develop proof systems that allow us to gain more structural insight into the relevant philosophical concepts. It turns out that this is not an altogether straightforward task—but in this talk, I hope to make at least some progress by analyzing strategies that have worked for certain hyperintensional logics.

Wednesday 22 September

9.30-10.10 Arnon Avron, "The Active Role of Language Extensions in Mathematical Reasoning"

ABSTRACT Extending the language of a theory **T** by new predicate and function symbols is usually not considered to be an essential component of the reasoning from **T**, but a matter of convenience, justified by the extension-by-definitions procedure or sometimes by the process of skolemization.

In this talk we argue that actually there are important cases in mathematics in which a systematic process of repeatedly extending the base language of **T** is an essential ingredient of the reasoning from **T**. A particularly important case of this sort is that of *predicative* set theory. We show that the systematic use of predicatively justified introduction of new predicate and function symbols allows us to go well beyond Feferman-Schüte ordinal Γ_0, which is usually taken to be "the limit of predicativity".

10.20-11.00 Alex Paseau, "What is Formalisation?"

ABSTRACT Turning informal language into formal symbolism is the logician's art. But what is this art, skill, possibly science, logicians are so adept at? What criteria guide it? You might expect the question to have an easy answer given that logicians are as a group excellent formalisers (almost by definition). But practice is one thing and theory another, and logicians or philosophers of logic have so far not greatly clarified what formalisation consists in.

My talk's aim will be to take a step towards a more complete list of criteria of acceptable or good formalisation. It will thereby cast some light on the main function of a logic: capturing logical consequence. My talk will be self-contained but if anyone would like to read some related material on which it will build, they could look at two recent papers of mine: 'Capturing Consequence' (2019) or 'Propositionalism' (2021).

11.30-12.10 Amir Tabatabai, "Logic As The Shadow of Mathematics"

ABSTRACT In Brouwerian intuitionism, mathematics is the world of mental constructions and logic as the collection of the universal laws behind these constructions is nothing but a distorted shadow of the real mathematics. This role is clearly far from the foundational role that logic is usually believed to play. In this talk, we try to formalize this Brouwerian extrinsic interpretation of logic.

14.00–14.40 Carlo Nicolai. "Cuts and Truths: Cut Elimination and Disquotation"

ABSTRACT I discuss some results and open problems for the application of standard cut-elimination strategies to systems featuring rules that provide the equivalence (suitably regimented) of A and 'A' is true (and extensions thereof). Due to paradox, the systems need to be nonclassical. I will mostly focus on some sub-structural systems. The obvious problem is that the step from A to 'A' is true collapses the logical complexity of the formula.

There are several ways of overcoming the problem: I consider noncontractive systems where one can disregard the logical complexity of formulae, but new questions arise about the nature of quantifiers. Another option is to drop the structural rule of identity (aka reflexivity): an additional measure of the number of applications of the truth predicate can now be added, and the system enjoys both cut elimination and an intuitive semantics.

14.50–15.30 Takahiro Yamada, "A Formalisation of Crispin Wright's Strict Finitistic First-Order Logic"

ABSTRACT "Strict finitism" is a constructive view obtained from intuitionism by replacing the notion of "possibility in principle" on which intuitionism is based, with that of "possibility in practice". Among the literature, Wright [1982] is of special interest, as it contains (i) an informal strict finitistic argument about numbers, and (ii) a sketch of systems of strict finitistic reasoning, formalised in his strict finitistic metatheory. The argument is to establish that there is a bijection between $\{n \in \mathbb{N} | n \leq \sigma\}$ and $\{n \in \mathbb{N} | 1 \leq n \leq \sigma - 1\}$, where σ is a number, such as $1,000,000^{1,000,000}$, practically representable in some notation, but not in decimal notation. In our talk, we will present a reconstruction of his first-order logic in the classical metatheory, as a step towards formally representing such an argument.

We will provide a sound and complete pair of a Kripke-style semantics and a natural deduction. While Wright's original semantics is similar to that of **IQC**, we will use the existence predicate (E) as in **IQCE** (cf. e.g. Troelstra & van Dalen [1988]). This is to properly formalise quantification. The complication is brought mainly by strict finitistic negation. It stands for practical impossibility: $k \models \neg A$ iff $l \not\models A$ for all l. Hence $\neg P(a)$ can meaningfully hold at k even if object a is not in the domain of k. Thus quantification should range over the object in the whole frame if the term is within the scope of $\neg$; otherwise, it should be restricted. E will denote the object that "exist" or are "available" to the agent, in order to explicate this restriction. We will have the two modes of quantification at the same time.

The other connectives are rather faithfully interpreted from Wright's semi-formal definitions. His implication $A \to B$ means

that if A holds in the future, so does B: $k \models A \rightarrow B$ iff for any $k' \geq k$ with $k' \models A$, there is a $k'' \geq k'$ such that $k'' \models B$. This is intuitionistic implication with "time-gap". He did not restrict the length of the gap. We assume it was because every structure in his strict finitistic metatheory is considered "practically small enough". We would, as part of classical idealisation, also accept a gap of any finite length.

We can use our logic to formalise and analyse informal, strict finitistic concepts. One example is Wright's stipulation of the "weak decidability" principle that every formula must be either practically verifiable or not. $\neg A \vee \neg\neg A$ is valid in our reconstruction, and we will provide an explanation of why this can be regarded as a formalisation of said principle.

However, some of Wright's expectations are not met. For one, Modus Ponens does not hold in general, although it does under his nonstandard criterion. Another is what we call the "prevalence" of a formula. We call a formula A *prevalent* if for any k, there is a $k' \geq k$ with $k' \models A$. Wright rejected the principle that all satisfiable formulas are prevalent, as it is unnatural: the verification of a formula may as well require so many resources that it could not be verified after verifying others. We found, however, that this principle is equivalent to (i) $\neg\neg A \rightarrow A$ and to (ii) $(A \rightarrow B) \rightarrow ((A \rightarrow \neg B) \rightarrow \neg A)$ in our classical formalisation. Interestingly, Wright accepted (ii) as valid, in his strict finitistic metatheory.

Thursday 23 September

9.30-10.10 Michael De, "On the Semantics (vs Syntax) of Relevance"

ABSTRACT The motivation behind relevance logic is that an inference is valid just in case (i) it necessarily preserves truth, and (ii) the premises and conclusion are relevantly related. Thus, there are two ways in which an inference can go wrong, i.e. by committing a fallacy of relevance or by failing to preserve truth. This idea is made clear in Belnap and Anderson 1975 and reflected in the original deductive systems for relevance logic, where necessary truth preservation and relevance are kept neatly separate.

However, this neat separation has been lost with the advent of relational semantics for relevance logic, according to which an inference is good simpliciter just in case it necessarily preserves truth. This has led to various related objections to relevance logic on the grounds that "the radical case for relevance should be dismissed just because the hypothesis it requires us to entertain is inconsistent" (Lewis 1982). In this talk I argue that proof-theoretic treatments of relevance are preferable to semantic ones, but suggest a way of semantically treating relevance that keeps it separate from truth preservation.

10.20-11.00 Robert Passmann, "On the Logical Instability of Mathematical Theories"

ABSTRACT It is a well-known that the Axiom of Choice entails the Law of Excluded Middle in intuitionistic Zermelo-Fraenkel set theory (Diaconescu–Goodman–Myhill). This result is an example for how the logic of non-classical theories can be unstable: adding mathematical axioms may entail changes in the logic of a given theory. This phenomenon has been well-investigated as a mathematical phenomenon (in particular, in the meta-mathematics of intuitionistic theories; consider, e.g., the so-called De Jongh Theorems) but has received surprisingly little philosophical attention. I will analyse its philosophical fruitfulness for a range of philosophical debates: logical pluralism, (constructive) theory choice, and the demarcation between logic and mathematics.

11.30-12.10 Albert Visser, "There is No Minimal Essentially Undecidable Theory"

ABSTRACT In this talk, I explain a result obtained in collaboration with Fedor Pakhomov and Juvenal Murwanashyaka. The result tells us that there is no interpretability minimal theory among the essentially undecidable RE theories. The talk will give some background both on the question and on the various notions of undecidability. I will sketch the main ideas of the two arguments that we found to prove the result.