“Meaning and Interpretation in Mathematics”

Andrew Arana

Abstract

Interpretability in logic formalizes the notion of a “dictionary” for translating statements of one theory into statements of another, in such a way that provability is preserved. For instance, translating “geodesic on the pseudosphere” to “Euclidean straight line” leads by way of Beltrami’s theorem to the observation that hyperbolic geometry is interpretable in Euclidean geometry. Interpretability is thus a way to capture a notion of translation that preserves provability. One might seek to take the dictionary metaphor further and ask whether interpretability preserves meaning as well.

In this talk I will address this question by taking into account the constraint on mathematical proof known as “purity of methods”, the proof ideal that says that a proof of a theorem should avoid what is foreign or extraneous to that theorem. Following Hilbert, purity can be understood in terms of meaning, by what belongs to the content of the theorem being proved. Whether a given proof is pure, then, comes down to the meaning of what is proved. It is easy to find algebraic statements interpretable in geometric theories and vice-versa. If meaning is preserved by interpretability, a pure proof of a geometric theorem could employ algebraic statements suitably interpreted in geometric terms. This would be to reject as empty the age-old attempts of mathematicians to develop the autarky of mathematical domains.

This talk’s general thesis is that while interpretability might seem to present difficulties for assessing purity attributions, one must be quite careful in the lessons that one draws from interpretability, and as a result these alleged difficulties are not so clear-cut.