"The Active Role of Language Extensions in Mathematical Reasoning"

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Abstract

Extending the language of a theory **T** by new predicate and function symbols is usually not considered to be an essential component of the reasoning from **T**, but a matter of convenience, justified by the extension-by-definitions procedure or sometimes by the process of skolemization.

In this talk we argue that actually there are important cases in mathematics in which a systematic process of repeatedly extending the base language of **T** is an essential ingredient of the reasoning from **T**. A particularly important case of this sort is that of *predicative* set theory. We show that the systematic use of predicatively justified introduction of new predicate and function symbols allows us to go well beyond Feferman-Schüte ordinal Γ_0, which is usually taken to be "the limit of predicativity".