"On the Logical Instability of Mathematical Theories"

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Abstract

It is a well-known that the Axiom of Choice entails the Law of Excluded Middle in intuitionistic Zermelo-Fraenkel set theory (Diaconescu–Goodman–Myhill). This result is an example for how the logic of non-classical theories can be unstable: adding mathematical axioms may entail changes in the logic of a given theory. This phenomenon has been well-investigated as a mathematical phenomenon (in particular, in the meta-mathematics of intuitionistic theories; consider, e.g., the so-called De Jongh Theorems) but has received surprisingly little philosophical attention. I will analyse its philosophical fruitfulness for a range of philosophical debates: logical pluralism, (constructive) theory choice, and the demarcation between logic and mathematics.