“A Formalisation of Crispin Wright’s Strict Finitistic First-Order Logic”

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**Abstract**

“Strict finitism” is a constructive view obtained from intuitionism by replacing the notion of “possibility in principle” on which intuitionism is based, with that of “possibility in practice”. Among the literature, Wright [1982] is of special interest, as it contains (i) an informal strict finitistic argument about numbers, and (ii) a sketch of systems of strict finitistic reasoning, formalised in his strict finitistic metatheory. The argument is to establish that there is a bijection between $\{n \in \mathbb{N} | n \leq \sigma\}$ and $\{n \in \mathbb{N} | 1 \leq n \leq \sigma - 1\}$, where $\sigma$ is a number, such as $1,000,000^{1,000,000}$, practically representable in some notation, but not in decimal notation. In our talk, we will present a reconstruction of his first-order logic in the classical metatheory, as a step towards formally representing such an argument.

We will provide a sound and complete pair of a Kripke-style semantics and a natural deduction. While Wright’s original semantics is similar to that of **IQC**, we will use the existence predicate ($E$) as in **IQCE** (cf. e.g. Troelstra & van Dalen [1988]). This is to properly formalise quantification. The complication is brought mainly by strict finitistic negation. It stands for practical impossibility: $k \models \neg A$ iff $l \not\models A$ for all $l$. Hence $\neg P(a)$ can meaningfully hold at $k$ even if object $a$ is not in the domain of $k$. Thus quantification should range over the object in the whole frame if the term is within the scope of $\neg$; otherwise, it should be restricted. $E$ will denote the object that “exist” or are “available” to the agent, in order to explicate this restriction. We will have the two modes of quantification at the same time.

The other connectives are rather faithfully interpreted from Wright’s semi-formal definitions. His implication $A \to B$ means that if $A$ holds in the future, so does $B$: $k \models A \to B$ iff for any $k' \geq k$ with $k' \models A$, there is a $k'' \geq k'$ such that $k'' \models B$. This is intuitionistic implication with “time-gap”. He did not restrict the length of the gap. We assume it was because every structure in his strict finitistic metatheory is considered “practically small enough”. We would, as part of classical idealisation, also accept a gap of any finite length.

We can use our logic to formalise and analyse informal, strict finitistic concepts. One example is Wright’s stipulation of the “weak decidability” principle that every formula must be either practically verifiable or not. $\neg A \vee \neg\neg A$ is valid in our reconstruction, and we will provide an explanation of why this can be regarded as a formalisation of said principle.

However, some of Wright’s expectations are not met. For one, Modus Ponens does not hold in general, although it does under his non-standard criterion. Another is what we call the “prevalence” of a formula. We call a formula $A$ *prevalent* if for any $k$, there is a $k' \geq k$ with $k' \models A$. Wright rejected the principle that all satisfiable formulas are prevalent, as it is unnatural: the verification of a formula may as well require so many resources that it could not be verified after verifying others. We found, however, that this principle is equivalent to (i) $\neg\neg A \to A$ and to

(ii) $(A \to B) \to ((A \to \neg B) \to \neg A)$ in our classical formalisation. Interestingly, Wright accepted (ii) as valid, in his strict finitistic metatheory.